Numerical Methods for Partial Differential Equations

ECTS: 6 ECTS

COORDINATOR: Generosa Fernández Manín (manin@dma.uvigo.es)

UNIVERSITY WHERE THE COORDINATOR IS: UVigo

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

LECTURER 1: Guillermo García Lomba (guille@dma.uvigo.es)

UNIVERSITY WHERE THE LECTURER 1 IS: UVigo

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

LECTURER 2: Laura Saavedra Lago (laura.saavedra@upm.es)

UNIVERSITY WHERE THE LECTURER 2 IS: UPM

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

SUBJECT CONTENTS

-Introduction to the numerical methods for the resolution of Differential Equations: finite differences, finite elements, finite volumes (3h).

-Methods of finite differences and finite elements in one dimensional problems (9h).

-Methods of finite differences and finite elements in several dimensions: elliptical, parabolic and hyperbolic problems (18h).

-Interactive classes using COMSOL-MULTIPHYSICS (12h).

METHODOLOGY

-Troubleshooting and / or exercises: the student has to resolve and deliver theoretical exercises of compression of the methods, practical of application to concrete problems and resolved with some software of numerical simulation: Matlab or Comsol Multiphysics.

-Practice in computer rooms: in the computer laboratory and using COMSOL Multiphysics resolve real cases simplified of diverse subjects: thermal, linear elasticity, electromagnetism, etc.

-Master sessions: these classes are devoted to explain the theoretical contents, to resolve some exercise to understand the methods and to introduce the practices of laboratory.

LANGUAGE USED IN CLASS: Spanish

IS IT COMPULSORY TO ATTEND CLASS? Students can attend via conference system

BIBLIOGRAPHY

-LeVeque,R.J., Finite Difference Methods for Ordinary and Partial Differential Equations: Steady State and Time Dependent Problems, SIAM,2007.

-Samarskii, A.A., The Theory of Difference Schemes,Marcel Dekker, New York, 2001.

Strickwerda, J.C., Finite Difference Schemes and Partial Differential Equations,Chapman &Hall/CRC, Boca Raton, 1999.

-Reddy, J.N., An introduction to the Finite Element Method, 2ª y 3ª(1993 y 2006), Mc Graw Hill.

-Johnson, C., Numerical solution for partial differential equations, 2009, Dover publications

-Eriksson, K. Estep, D. Hansbo, P. Johnson, C., Computational differential equations, 1996, Cambridge.

-Class notes and COMSOL MULTIPHYSICS manuals.

SKILLS

Basic:

CG2: To be able to apply the acquired knowledge and abilities to solve problems in new or unfamiliar environments within broader contexts, including the ability to integrate multidisciplinary R & D in the business environment.

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

Specific:

CE4: To be able to select a set of numerical techniques, languages and tools, appropriate to solve a mathematical model.

Numerical simulation specialization:

CS1: To know, be able to select or use how to handle most suitable professional software tools (both commercial and free) for the simulation of processes in the industrial and business sector.

WILL YOU BE USING A VIRTUAL PLATFORM? Yes. faitic.uvigo.es

WILL YOU BE USING ANY SPECIFIC SOFTWARE? Yes. COMSOL MULTIPHYSICS

CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

Evaluation:

1. Attendance and class participation (5%).
2. Individual exercises (25%).
3. Two lab practices (30% all the same).
4. Compulsory final exam: theory (20%) and lab practices (20%).

CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

Students who had followed the continuous evaluation will hand in the individual exercises and redo the final exam.

Students who were not able to follow the continuous evaluation will take a final exam which will include all the contents of the subject. This exam will be taken with no help materials, will be longer than the 1st assessment opportunity one and will have a different structure.

FURTHER COMMENTS:

The COMSOL practices will take place at the University of Vigo for students enrolled in the Galician universities. For students enrolled in the universities in Madrid, these practices will take place in Madrid and will be taught by Laura Saavedra Lago and Fernando Varas Mérida.

Numerical Methods and Programming

ECTS: 6 ECTS

COORDINATOR: Francisco José Pena Brage (fran.pena@usc.es)

UNIVERSITY WHERE THE COORDINATOR IS: USC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

LECTURER 1: José Antonio García Rodríguez (jagrodriguez@udc.es)

UNIVERSITY WHERE THE LECTURER 1 IS: UDC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

LECTURER 2: Duarte Santamarina Ríos (duarte.santamarina@usc.es)

UNIVERSITY WHERE THE LECTURER 2 IS: USC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

SUBJECT CONTENTS

Part I: Introduction to Programming

1. Introduction to Matlab. Basic commands and functions.

2. Vectors and Matrices in Matlab. Sparse matrices. Plots.

3. Files. M and programming. Data Structures in Matlab.

4. Introduction to Fortran 90: Data types and control flow.

5. Arrays in Fortran 90. Procedures, modules and interfaces.

6. Input / Output data in Fortran 90.

Part II. numerical Methods

7. Numerical solution of systems of linear equations: Condition of a system of linear equations. Direct methods: LU, LL^t, LDL^t and QR. Classical iterative methods: Jacobi, Gauss-Seidel, SOR and SSOR. Convergence criteria.

8. Numerical solution of systems of non-linear equations: Review of methods for solving non-linear equations. Fixed-point iteration. Newton's method. Computational considerations.

9. Interpolation, numerical differentiation and integration: Lagrange interpolation. Hermite interpolation. Runge effect. Spline approximation. Numerical differentiation of polynomial interpolating type. Numerical quadrature of polynomial interpolating type. Newton-Cotes formulas. Gauss formulas. Composite quadrature.

9. Interpolation. Lagrange interpolation. Hermite interpolation. Runge effect. Spline approximation.

10. Numerical differentiation and integration. Numerical differentiation of polynomial interpolating type. Numerical quadrature of polynomial interpolating type in a single variable. Newton-Cotes formulas. Gauss formulas. Composite quadrature.

11. Numerical interpolation and integration in several variables.

METHODOLOGY

Theory will be taught in order for students to build small computer programs under guidance as an introduction to programming. Students will also carry out other tasks by themselves to strengthen their knowledge.

Students will work individually on numerical methods in order to deepen their knowledge on the subject.

LANGUAGE USED IN CLASS: It will depend on the audience.

IS IT COMPULSORY TO ATTEND CLASS? Students can attend via conference system.

BIBLIOGRAPHY

Basic bibliography:

T. Aranda, J.G. García, Notas sobre Matlab. Universidad de Oviedo, Servicio de Publicaciones, 1999.

J.F. Epperson. An introduction to numerical methods and analysis. Edición revisada. John Wiley & Sons, 2007.

M. Metcalf, J.K. Reid. Modern Fortran Explained Oxford University Press, 2011.

Additional bibliography:

S.J. Chapman, Fortran 90/95 for scientists and engineers. WCB/McGrawHill, 2004.

P.G. Ciarlet. Introducción á análise numérica matricial e á optimización. Universidade de Santiago, 2011.

J.D. Faires, R. Burden. Análisis Numérico. Thomson 2011.

G.H. Golub, C.F. van Loan, Matrix Computations. John Hopkins, University Press, 1996.

MathWorks Programming Guide in Matlab:
http://www.mathworks.com/access/helpdesk/help/techdoc/matlab_prog/matlab_prog.html

D.C. Hanselman, B.L. Littlefield. Mastering Matlab 7. Prentice Hall, 2004.

J.A. Infante del Río, J.M. Rey Cabezas, Métodos numéricos: teoría, problemas y prácticas con Matlab.Piramide, 2007.

C.T. Kelley. Solving Nonlinear Equations with Newton's Method. SIAM, 2003.

D. Kincaid, W. Cheney. Análisis numérico. Las matemáticas del cálculo científico. AddisonWesley Iberoamericana, 1994.

J.H. Mathews, K.D. Fink, Métodos Numéricos con Matlab. Prentice Hall, 2000.

M. Metcalf, J.K. Reid. Fortran 90/95 explained. Oxford University Press, 1999.

W.H. Press. Numerical Recipes in Fortran 90: Volume 2. Cambridge University Press, 1996.

A. Quarteroni, F. Saleri. Cálculo Científico con MATLAB y Octave. Springer, 2006.

J.M. Viaño, M. Burguera. Lecciones de métodos numéricos. 3. Interpolación. Tórculo Edicións, 1999.

J.M. Viaño. Lecciones de métodos numéricos. 2. Resolución de ecuaciones numéricas. Tórculo Edicións, 19

SKILLS

Basic:

CG2: To be able to apply the acquired knowledge and abilities to solve problems in new or unfamiliar environments within broader contexts, including the ability to integrate multidisciplinary R & D in the business environment;

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

Specific:

CE4: To be able to select a set of numerical techniques, languages and tools, appropriate to solve a mathematical model.

Numerical Simulation specialization:

CS2: To adapt, modify and implement software tools for numerical simulation.

WILL YOU BE USING A VIRTUAL PLATFORM? Yes, the Google group called "Métodos Numéricos y Programación (M2I)". Each student must access to his Google account and, from there, "Ask membership in the forum" in https://groups.google.com/d/forum/mnp-m2i. The application must indicate a public name and the rest of the required information that appears in the dialog.

WILL YOU BE USING ANY SPECIFIC SOFTWARE? Yes. Matlab GNU Fortran

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CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

The first part (50% of the qualification) will consist on the evaluation of the Matlab and Fortran practical works; both works will have the same weight to calculate the qualification of this part.

The second part (the remaining 50%) will correspond to the exam, where the concepts acquired in the part II of the subject will be evaluated.

Students must pass both parts in order to pass the subject. If one of the parts is not passed the qualification will be 4 out of 10.

CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

The same as for the first opportunity. The deadline for handing in the tasks will be adapted to the date of the second exam.

Ordinary Differential Equations / Dynamical Systems

ECTS: 6 ECTS

COORDINATOR: Óscar López Pouso (oscar.lopez@usc.es)

UNIVERSITY WHERE THE COORDINATOR IS: USC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

LECTURER 1: Jerónimo Rodríguez García (jeronimo.rodriguez@usc.es)

UNIVERSITY WHERE THE LECTURER 1 IS: USC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

SUBJECT CONTENTS

I. NUMERICAL METHODS FOR INITIAL VALUE PROBLEMS (IVP) ASSOCIATED TO ORDINARY DIFFERENTIAL EQUATIONS (ODEs):

1. Concept of initial value problem for ODEs. Concept of numerical method to approximate the solution of that problem.
2. Description of Euler methods: explicit (forward) and implicit (backward).
3. Definition of convergence and order of convergence. Discretization error and rounding error, effect of rounding errors on convergence.
4. Concept of multistep method, compared to one-step method. For multistep methods: concept of starting procedure, method for making the starting procedure, and order theorem for that method.
5. One-step non-linear methods of high order: Runge-Kutta methods (RK) (description).
6. Linear multistep methods (LMM) of high order (description):

a LMM based on numerical quadrature:

i. Adams Bashforth family.

ii. Adams Moulton family.

iii. Nyström family.

iv. Milne Simpson family.

b. LMM based on numerical differentiation: BDF methods.

7. MATLAB® commands for solving ODEs.

II. DYNAMICAL SYSTEMS:

1. Linear dynamical systems.

a. Linear vector fields.

b. Calculus of the exponential of a matrix. Jordan canonical form.

c. Teorema fundamental de existencia y unicidad de solución para sistemas lineales.

d. Invariant subspaces: stable, unstable and central spaces.

2. Basic theorems related to the general theory of differential equations.

a. The fundamental theorem of existence and uniqueness of solution. Dependence on the parameters and on the initial conditions.

b. The problem of extension of solutions. Maximal solutions

c. Flux associated to a differential field. Singular and regular points. Orbits. α-limit and ω-limit sets.

3. Local theory.

a. Liapunov stability. Liapunov functions.

b. Concepts of equivalence and topological conjugacy. Structural stability.

c. The invariant manifold theorem.

d. The Hartman Grobman theorem.

e. Gradient and Hamiltonian systems.

4. Global theory.

a. The concept of limit cycle.

b. Electric circuits. Lienard systems. The Van der Pol equation.

c. The Poincaré map.

5. Introduction to the bifurcation theory.

METHODOLOGY

1. Planning for the contents of each class.
2. Explanation on blackboard (lecture) or equivalent by using videoconferencing.
3. Programming some methods on the computer.

LANGUAGE USED IN CLASS: Spanish

IS IT COMPULSORY TO ATTEND CLASS? It is not compulsory

BIBLIOGRAPHY

I. NUMERICAL METHODS FOR INITIAL VALUE PROBLEMS (IVP) ASSOCIATED TO ORDINARY DIFFERENTIAL EQUATIONS (ODES):

BASIC BIBLIOGRAPHY:

1. ASCHER, URI M.; PETZOLD, LINDA R. (1998) Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. SIAM, Philadelphia, PA.
2. HAIRER, ERNST; NØRSETT, SYVERT PAUL; WANNER, GERHARD (1987) Solving Ordinary Differential Equations I. Nonstiff Problems. Springer, Berlin.
3. ISAACSON, EUGENE; KELLER, HERBERT BISHOP (1994, unabridged, corrected republication) Analysis of Numerical Methods. Dover Publications, New York, NY. [Original edition: Wiley, 1966].
4. ISERLES, ARIEH (2008, second edition) A first course in the numerical analaysis of differential equations. Cambridge Texts in Applied Mathematics. Cambridge University Press. Cambridge. [Primera edición: 1997]
5. LAMBERT, JOHN DENHOLM (1991) Numerical Methods for Ordinary Differential Systems. Wiley, Chichester.
6. STOER, JOSEF; BULIRSCH, ROLAND (1993, second edition) Introduction to Numerical Analysis. Springer, New York, NY. [First edition: 1980].

<u>COMPLEMENTARY BIBLIOGRAPHY:</u>

1. BUTCHER, JOHN CHARLES (2008, segunda edición) Numerical Methods for Ordinary Differential Equations. Wiley, Chichester. [First edition: 2003]
2. CROUZEIX, MICHEL; MIGNOT, ALAIN L. (1989, second edition) Analyse Numérique des Équations Différentielles. Masson, Paris. [First edition: 1984].

3. DEKKER, KEES; VERWER, JAN G. (1984) Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations. Elsevier Science Publishers B. V., Amsterdam.

4. HAIRER, ERNST; WANNER, GERHARD (1991) Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems. Springer, Berlin.

5. HENRICI, PETER (1962) Discrete Variable Methods in Ordinary Differential Equations. Wiley. New York, NY.

6. KINCAID, DAVID RONALD; CHENEY, ELLIOT WARD (1991) Numerical Analysis. Brooks/Cole, Pacific Grove, CA.

7. LAMBERT, JOHN DENHOLM (1973) Computational Methods in Ordinary Differential Equations. Wiley, London.

8. QUARTERONI, ALFIO; SACCO, RICCARDO; SALERI, FAUSTO (2000) Numerical Mathematics. Springer, New York, NY.

II. **DYNAMICAL SYSTEMS**:

BASIC BIBLIOGRAPHY:

1. Lawrence Perko. Differential Equations and Dynamical Systems. Texts in Applied Mathematics 7. Springer. Third edition. 2000.

2. Morris W. Hirsch, Stephen Smale. Differential Equations, Dynamical Systems and Linear Algebra. Pure and Applied Mathematics. Academic Press. 1974.

COMPLEMENTARY BIBLIOGRAPHY:

1. John Guckenheimer, Philip Holmes. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer-Verlag New York. 1983.

2. Jack K. Hale, Hüseyin Koçak. Dynamics and Bifurcations. Springer-Verlag New York. 1991.

3. Richard H. Enns, George C. McGuire. Computer Algebra Recipes. An Advance Guide to Scientific Modeling. Springer. 2007.

SKILLS

Basic:

CG1: To have knowledge that provide a basis or opportunity for originality in developing and / or applying ideas, often within a research context, knowing how to translate industrial needs in terms of R & D in the field of mathematics Industrial.

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

Specific:

CE3: To determine if a model of a process is well made and well mathematically formulated from a physical standpoint.

Modelling specialization:

CM1: To be able to extract, using different analytical techniques, both qualitative and quantitative models.

WILL YOU BE USING A VIRTUAL PLATFORM? Yes. Moodle (USC)

WILL YOU BE USING ANY SPECIFIC SOFTWARE? Yes. MATLAB and MAPLE.

CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

To pass the course it is compulsory to hand in the exercises and programming practices commissioned by the teachers within the timeframes set to it. The final grade will result from a written examination in which the part devoted to programming practices will weigh 30% of the total.

CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

The same criteria as the ones used in the first assessment opportunity.

FURTHER COMMENTS:

Teachers will teach in English if need to.

Partial Differential Equations

ECTS: 6 ECTS

COORDINATOR: José Durany Castrillo (durany@dma.uvigo.es)

UNIVERSITY WHERE THE COORDINATOR IS: UVigo

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? No

LECTURER 1: Fernando Varas Mérida (fernando.varas@upm.es)

UNIVERSITY WHERE THE LECTURER 1 IS: UPM

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? No

SUBJECT CONTENTS

1. Classical analysis of linear partial differential equations.

   a) Classic examples: the equations of Laplace, heat and wave.

   b) Classification of linear partial differential equations.

   c) Existence and uniqueness.

   d) Study of analytical techniques of resolution: the Laplace equation in a circle, in a ring and in a box.

   e) The heat equation in an isolated finite bar, non-isolated and general case.

   f) The wave equation in an isolated finite string, non-isolated and general case.

2. Variational formulation of elliptic problems, linear elasticity and Stokes system.

3. Introduction to the variational formulation of evolutionary problems: parabolic and hyperbolic problems.

www.m2i.es
info@m2i.es

METHODOLOGY

1) Master sessions: the contents of the subject will be given in these sessions.

2) Formulation, analysis and resolution of problems and exercises related to the subject.

Classes via videoconferencing system.

LANGUAGE USED IN CLASS: Spanish

IS IT COMPULSORY TO ATTEND CLASS? Students can attend via conference system.

BIBLIOGRAPHY

Brezis, Analyse fonctionelle. Masson, 1983.

E. Casas, Introducción a las ecuaciones en derivadas parciales. Univ. Cantabria, 1992.

E. di Benedetto, Partial differential equations. Birkhauser, 1995.

D. Gilbarg, N.S. Trudinger, Elliptic partial differential equations of second order. Springer, 1983.

J.L. Lions, Quelques methodes de resolution des problemes aux limites non lineaires. Dunod, 1969.

V.P. Mijailov, Ecuaciones diferenciales en derivadas parciales. MIR-Moscú, 1976.

J. Necas, Les methodes directes en theorie des equations elliptiques. Masson, 1967.

I. Peral, Primer curso de ecuaciones en derivadas parciales. Addison-Wesley. Univ. Autónoma Madrid, 1995.

P.A. Raviart, J.M. Thomas, Introduction a l'analyse numerique des equations aux derivees partielles. Masson, 1983.

Showalter, R. E., Monotone Operators in Banach Space and Nonlinear Partial Differential Equations. Mathematical Surveys and Monographs Volume 49. American Mathematical Society (AMS), 1997. (Chapter I & II)

R. Temam, Navier-Stokes equations. North-Holland, 1977.

SKILLS

Basic:

www.m2i.es
info@m2i.es

CG1: To have knowledge that provide a basis or opportunity for originality in developing and / or applying ideas, often within a research context, knowing how to translate industrial needs in terms of R & D in the field of mathematics Industrial.

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

Specific:

CE3: To determine if a model of a process is well made and well mathematically formulated from a physical standpoint.

Modelling specialization:

CM1: To be able to extract, using different analytical techniques, both qualitative and quantitative models.

WILL YOU BE USING A VIRTUAL PLATFORM? Yes. faitic.uvigo.es

WILL YOU BE USING ANY SPECIFIC SOFTWARE? No.

CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

The assesment will take into account:

1) individual exercises that will result in 60% of the grade.

2) a test that will result in 40% of the grade.

CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

Same as the 1st assessment opportunity.