

## Variational analysis of partial differential equations

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**ECTS:** 3 ECTS

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**COORDINATOR:** Rafael Muñoz Sola ([rafael.munoz@usc.es](mailto:rafael.munoz@usc.es))

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**UNIVERSITY WHERE THE COORDINATOR IS:** USC

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**HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES?** No

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### SUBJECT CONTENTS

1, Variational inequalities.

1.1. Variational inequalities: introduction (the obstacle problem).

1.2. Theorems of existence and uniqueness of solution of variational inequalities.

1.3. Applications.

2. Eigenfunctions and spectral decomposition.

2.1. Introduction to spectral problems.

2.2. Theorems of existence of eigenvalues and eigenvectors for an abstract spectral problem.

2.3. Applications to elliptic boundary value problems.

3. Variational theory for linear evolution problems.

3.1. Parabolic problems.

3.1.1. Weak formulation.

3.1.2. Inequality of the energy.

3.1.3. Uniqueness of the solution. Continuous dependency of the solution respect to the data.

3.2. Introduction to hyperbolic problems of order 2 in time.

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## METHODOLOGY

Theoretical contents and exercises adapted to the goals of the course will be presented.

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**LANGUAGE USED IN CLASS:** Spanish

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**IS IT COMPULSORY TO ATTEND CLASS?** Students can attend via conference system.

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## BIBLIOGRAPHY

- Basic bibliography:

[1] BRÉZIS, HAÏM. Analyse fonctionnelle. Théorie et applications. Collection Mathématiques Appliquées pour la Maîtrise. Masson, Paris, 1983. [Traducción al castellano: Análisis funcional. Teoría y aplicaciones. Alianza Universidad Textos. Alianza Editorial, S.A., Madrid, 1984].

[2] CASAS RENTERÍA, EDUARDO. Introducción a las ecuaciones en derivadas parciales. Cantabria: Servicio de Publicaciones, Universidad, D.L., 1992.

[3] EVANS, LAWRENCE CRAIG. Partial differential equations. Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 1998.

[4] GLOWINSKI, ROLAND. Numerical methods for nonlinear variational problems. Springer Series in Computational Physics. Springer, New York, 1984.

[5] LIONS, JACQUES-LOUIS. Contrôle optimal de systèmes gouvernés par des équations aux dérivées partielles. Dunod, Paris, 1968.

[6] RAVIART, PIERRE-ARNAUD; THOMAS, JEAN-MARIE. Introduction à l'analyse numérique des équations aux dérivées partielles. Collection Mathématiques Appliquées pour la Maîtrise. Masson, Paris, 1983.

- Complementary bibliography:

[7] CHIPOT, MICHEL. Elements of nonlinear analysis. Birkhäuser, Basel, 2000.

[8] DAUTRAY, ROBERT; LIONS, JACQUES-LOUIS. Mathematical analysis and numerical methods for science and technology. Vols. 1-6. Springer, Berlin, 1990-1993.

[9] EKELAND, IVAR; TEMAM, ROGER. Analyse convexe et problèmes variationnels. Collection Études Mathématiques. Dunod; Gauthier-Villars, Paris-Brussels-Montreal, 1974.[ Traducción al inglés: Convex analysis and variational problems, SIAM, Filadelfia, 1999.]

[10] KINDERLEHRER, DAVID; STAMPACCHIA, GUIDO. An introduction to variational inequalities and their applications. Pure and Applied Mathematics, 88. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York-London, 1980.

[11] LIONS, JACQUES-LOUIS. Quelques méthodes de résolution des problèmes aux limites non linéaires. Dunod, Paris, 1969.

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[12] SHOWALTER, RALPH EDWIN. Monotone operators in Banach space and nonlinear partial differential equations. Mathematical Surveys and Monographs, Vol. 49, American Mathematical Society, Providence (Rhode Island), 1997.

[13] TEMAM, ROGER. Infinite-dimensional dynamical systems in Mechanics and Physics. Applied Mathematical Sciences, 68, Springer, New York, 1997 (segunda edición; primera edición de 1988).

[14] VIAÑO REY, JUAN MANUEL. Inecuaciones variacionales: teoría y algoritmos. Tesina de licenciatura, Dpto. de Ecuaciones Funcionales, Univ. de Santiago de Compostela, 1978.

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## SKILLS

### Basic:

CG1: To have knowledge that provide a basis or opportunity for originality in developing and / or applying ideas, often within a research context, knowing how to translate industrial needs in terms of R & D in the field of mathematics Industrial.

CG3: To be able to integrate knowledge in order to state opinions using information that even incomplete or limited, include reflecting on social and ethical responsibilities linked to the application of their knowledge.

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

### Specific:

CE3: To determine if a model of a process is well made and well mathematically formulated from a physical standpoint.

CE5: To be able to validate and interpret the results, comparing them with visualizations, experimental measurements and functional requirements of the physical engineering system.

### Modelling specialization

CM1: To be able to extract, using different analytical techniques, both qualitative and quantitative models.

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**WILL YOU BE USING A VIRTUAL PLATFORM?** Yes. Moodle (USC)

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**WILL YOU BE USING ANY SPECIFIC SOFTWARE?** No.

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## CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

The evaluation in the first assessment opportunity will consist in two parts:

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- a final written examination, in which the knowledge, skills and abilities acquired along the course will be globally evaluated.

- Continuous evaluation of the student's work; this may include the evaluation of exercises and/or practices, as well as the essays.

The student who does not attend the final exam will be marked as "NO PRESENTADO" ("Failure to complete the final exam").

The final examination will account for 60% of the global mark of the subject.

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## CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

The evaluation in the first assessment opportunity will consist in two parts:

- a final written examination, in which the knowledge, skills and abilities acquired along the course will be globally evaluated.

- Continuous evaluation of the student's work; this may include the evaluation of exercises and/or practices, as well as the essays.

In order to carry out the continuous evaluation of the second assessment opportunity, the teacher will determine a new deadline for the delivery of exercises, practices and / or development of essays.

The student can keep for the second assessment opportunity the grade of the continuous evaluation that s/he has obtained in the first assessment opportunity.

The student who fails to complete the final exam of the first assessment opportunity and also fails to complete the final exam will be marked as "NO PRESENTADO" ("Failure to complete the final exam").

The final examination will account for 60% of the global mark of the subject.

If a student fails at the first assessment opportunity and sits the exam again, his/her final grade will be the highest of the two final grades.

If a student fails at the first assessment opportunity and does not sit the exam, his/her final grade will be the one obtained at the first opportunity.

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## FURTHER COMMENTS:

It is advisable to attend this subject: to have basic knowledge of Functional Analysis; to know the contents corresponding to the subject "Ecuaciones en Derivadas Parciales" (Partial Differential Equations) or to simultaneously attend it.

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