

## Advanced Finite Elements

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**ECTS:** 3 ECTS

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**COORDINATOR:** Jerónimo Rodríguez García ([jeronimo.rodriguez@usc.es](mailto:jeronimo.rodriguez@usc.es))

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**UNIVERSITY WHERE THE COORDINATOR IS:** USC

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**HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES?** Yes

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### SUBJECT CONTENTS

1. Approximation of elliptic problems Abstract: Lax-Milgram Lemma, Lemma of Cea.
2. Approximation of elliptic problems of order 2 in dimension 2 and 3 with Lagrange finite elements (triangles, tetrahedra, quadrilaterals and hexahedral): description and construction of finite element spaces, references, basic functions, affine equivalence.
3. A priori error estimates for affine equivalent families of finite element method: quality of meshes, convergence, regular families. Case of curved domains.
4. Computer programming method: elementary matrices and second members, quadrature, assembly, storage profile, boundary conditions. Applications to flexion of membranes, heat conduction, two- and three-dimensional elasticity.
5. Evolution parabolic problem and hyperbolic of order 2 in time: variational formulation, discretization in space and time.
6. Finite elements for fourth order problems: bending of elastic beam, elastic plate bending.
7. Introduction to mixed problems: Stokes equation. Existence and uniqueness of solution of the abstract formulation: inf-sup condition.
8. Mixed finite elements: numerical resolution of Stokes equation. A priori error estimates. Discrete inf-sup condition.

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### METHODOLOGY

The course is taught in the classroom using audiovisual materials and students will be provided with the teacher notes. Students in groups of 2 / 3 will carry out supervised essays on the resolution of different problems using finite elements.

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LANGUAGE USED IN CLASS: Spanish

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IS IT COMPULSORY TO ATTEND CLASS? Students can attend via conference system

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## BIBLIOGRAPHY

### BASIC BIBLIOGRAPHY:

Bécache, E., Ciarlet, P. J., Hazard, C., Luneville, E., La méthode des éléments finis: de la théorie a la pratique. Tome II. Compléments., Les Cours, Les Presses de l'ENSTA, Paris, 2010.

Ciarlet, P.G., Basic error estimates for elliptic problems. Handbook of Numerical Analysis. Vol. II. North Holland. 1991.

Ciarlet, P. J., Luneville, E., La méthode des éléments finis: de la théorie a la pratique. Tome I. Concepts généraux., Les Cours, Les Presses de l'ENSTA, Paris, 2009.

Krizek, M., Neittaanmaki, P., Finite element approximation of variational problems and applications. Longman Scientific&Technical, 1984.

Raviart, P.A., Thomas, J.M., Introduction à l'analyse numérique des équations aux dérivées partielles. Masson. 1983.

Viaño, J.M., Figueiredo, J., Implementação do método de elementos finitos. Notas dos autores. 2000.

### COMPLEMENTARY BIBLIOGRAPHY:

Brenner, S.C., Scott, L.R., The mathematical theory of finite element methods. Springer - Verlag. 1994.

Brezzi, F., Fortin, M., Mixed and hybrid finite element methods, vol. 15 of Springer Series in Computational Mathematics, Springer - Verlag, New York, 1991.

Ern, A., Guermond, J.L., Theory and Practice of finite elements. Springer - Verlag. 2004.

Girault, V., Raviart, P.A., Finite element methods for Navier - Stokes equations. Springer - Verlag. 1986.

Glowinski, R, Numerical methods for nonlinear variational problems. Springer. 1984.

Pironneau, O., Finite element methods for fluids. John Wiley - Masson. 1989.

Quarteroni, A., Numerical models for differential problems. Springer - Verlag. 2009.

Quarteroni, A., Valli, A., Numerical approximation of Partial Differential Equations. Springer - Verlag. 1997.

Roberts, J.E., Thomas, J.M., Mixed and hybrid methods. Handbook of Numerical Analysis. Vol . II. North Holland. 1991.

Thomee, V., Galerkin finite element methods for parabolic problems. Springer - Verlag. 1997.

Verfurth, R., A Review of A Posteriori Error Estimation and Adaptive Mesh - refinement Technique, Wiley & Teubner, 1996.

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## SKILLS

### Basic:

CG3: To be able to integrate knowledge in order to state opinions using information that even incomplete or limited, include reflecting on social and ethical responsibilities linked to the application of their knowledge.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

### Specific:

CE4: To be able to select a set of numerical techniques, languages and tools, appropriate to solve a mathematical model.

### Numerical simulation specialization:

CS2: To adapt, modify and implement software tools for numerical simulation.

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**WILL YOU BE USING A VIRTUAL PLATFORM?** Yes. Moodle (USC)

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**WILL YOU BE USING ANY SPECIFIC SOFTWARE?** Yes. MATLAB, Fortran

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## CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

Assessment: written exam of a selected part of the subject (50% of the final mark) and the essays carried out individually or in groups (50% of the final mark). To pass the subject is necessary to pass the two parts (ie, 2.5 points or more, over 5, both in the written examen as in the work). There are two exam opportunities in each course. The mark obtained in the essays will be kept for the second exam.

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## CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

The mark obtained in the essays will be kept for the second assessment opportunity.