www.m2i.es
info@m2i.es

Ordinary Differential Equations / Dynamical Systems

ECTS: 6 ECTS

COORDINATOR: Óscar López Pouso (oscar.lopez@usc.es)

UNIVERSITY WHERE THE COORDINATOR IS: USC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

LECTURER 1: Jerónimo Rodríguez García (jeronimo.rodriguez@usc.es)

UNIVERSITY WHERE THE LECTURER 1 IS: USC

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? Yes

SUBJECT CONTENTS

I. NUMERICAL METHODS FOR INITIAL VALUE PROBLEMS (IVP) ASSOCIATED TO ORDINARY DIFFERENTIAL EQUATIONS (ODEs):

1. Concept of initial value problem for ODEs. Concept of numerical method to approximate the solution of that problem.
2. Description of Euler methods: explicit (forward) and implicit (backward).
3. Definition of convergence and order of convergence. Discretization error and rounding error, effect of rounding errors on convergence.
4. Concept of multistep method, compared to one-step method. For multistep methods: concept of starting procedure, method for making the starting procedure, and order theorem for that method.
5. One-step non-linear methods of high order: Runge-Kutta methods (RK) (description).
6. Linear multistep methods (LMM) of high order (description):

    a LMM based on numerical quadrature:

    i. Adams Bashforth family.

www.m2i.es
info@m2i.es

ii. Adams Moulton family.

iii. Nyström family.

iv. Milne Simpson family.

b. LMM based on numerical differentiation: BDF methods.

7. MATLAB® commands for solving ODEs.

II. DYNAMICAL SYSTEMS:

1. Linear dynamical systems.

a. Linear vector fields.

b. Calculus of the exponential of a matrix. Jordan canonical form.

c. Teorema fundamental de existencia y unicidad de solución para sistemas lineales.

d. Invariant subspaces: stable, unstable and central spaces.

2. Basic theorems related to the general theory of differential equations.

a. The fundamental theorem of existence and uniqueness of solution. Dependence on the parameters and on the initial conditions.

b. The problem of extension of solutions. Maximal solutions

c. Flux associated to a differential field. Singular and regular points. Orbits. α-limit and ω-limit sets.

3. Local theory.

a. Liapunov stability. Liapunov functions.

b. Concepts of equivalence and topological conjugacy. Structural stability.

c. The invariant manifold theorem.

d. The Hartman Grobman theorem.

e. Gradient and Hamiltonian systems.

4. Global theory.

a. The concept of limit cycle.

b. The Poincaré Bendixon theorem.

c. Electric circuits. Lienard systems. The Van der Pol equation.

d. The Poincaré map.

5. Introduction to the bifurcation theory.

a. Elementary bifurcations: saddle-node, transcritical, pitchfork, hysteresis.

www.m2i.es
info@m2i.es

b. Hopf bifurcation.

METHODOLOGY

1. Planning for the contents of each class.
2. Explanation on blackboard (lecture) or equivalent by using videoconferencing.
3. Programming some methods on the computer.

LANGUAGE USED IN CLASS: Spanish

IS IT COMPULSORY TO ATTEND CLASS? It is not compulsory

BIBLIOGRAPHY

I. NUMERICAL METHODS FOR INITIAL VALUE PROBLEMS (IVP) ASSOCIATED TO ORDINARY DIFFERENTIAL EQUATIONS (ODES):

BASIC BIBLIOGRAPHY:

1. ASCHER, URI M.; PETZOLD, LINDA R. (1998) Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. SIAM, Philadelphia, PA.

1. HAIRER, ERNST; NØRSETT, SYVERT PAUL; WANNER, GERHARD (1987) Solving Ordinary Differential Equations I. Nonstiff Problems. Springer, Berlin.

2. HENRICI, PETER (1962) Discrete Variable Methods in Ordinary Differential Equations. Wiley, New York, NY.

3. ISAACSON, EUGENE; KELLER, HERBERT BISHOP (1994, unabridged, corrected republication) Analysis of Numerical Methods. Dover Publications, New York, NY. [Original edition: Wiley, 1966].

4. LAMBERT, JOHN DENHOLM (1991) Numerical Methods for Ordinary Differential Systems. Wiley, Chichester.

5. STOER, JOSEF; BULIRSCH, ROLAND (1993, second edition) Introduction to Numerical Analysis. Springer, New York, NY. [First edition: 1980].

COMPLEMENTARY BIBLIOGRAPHY:

1. BUTCHER, JOHN CHARLES (2003) Numerical Methods for Ordinary Differential Equations. Wiley, Chichester.

2. CROUZEIX, MICHEL; MIGNOT, ALAIN L. (1989, second edition) Analyse Numérique des Équations Différentielles. Masson, Paris. [First edition: 1984].

3. DEKKER, KEES; VERWER, JAN G. (1984) Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations. Elsevier Science Publishers B. V., Amsterdam.

4. HAIRER, ERNST; WANNER, GERHARD (1991) Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems. Springer, Berlin.

5. KINCAID, DAVID RONALD; CHENEY, ELLIOT WARD (1991) Numerical Analysis. Brooks/Cole, Pacific Grove, CA.

6. LAMBERT, JOHN DENHOLM (1973) Computational Methods in Ordinary Differential Equations. Wiley, London.

7. QUARTERONI, ALFIO; SACCO, RICCARDO; SALERI, FAUSTO (2000) Numerical Mathematics. Springer, New York, NY.

II. **DYNAMICAL SYSTEMS**:

BASIC BIBLIOGRAPHY:

1. Lawrence Perko. Differential Equations and Dynamical Systems. Texts in Applied Mathematics 7. Springer. Third edition. 2000.

2. Morris W. Hirsch, Stephen Smale. Differential Equations, Dynamical Systems and Linear Algebra. Pure and Applied Mathematics. Academic Press. 1974.

COMPLEMENTARY BIBLIOGRAPHY:

1. John Guckenheimer, Philip Holmes. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer-Verlag New York. 1983.

2. Jack K. Hale, Hüseyin Koçak. Dynamics and Bifurcations. Springer-Verlag New York. 1991.

3. Richard H. Enns, George C. McGuire. Computer Algebra Recipes. An Advance Guide to Scientific Modeling. Springer. 2007.

SKILLS

Basic:

CG1: To have knowledge that provide a basis or opportunity for originality in developing and / or applying ideas, often within a research context, knowing how to translate industrial needs in terms of R & D in the field of mathematics Industrial.

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

Specific:

CE3: To determine if a model of a process is well made and well mathematically formulated from a physical standpoint.

Modelling specialization:

CM1: To be able to extract, using different analytical techniques, both qualitative and quantitative models.

WILL YOU BE USING A VIRTUAL PLATFORM? Yes. Moodle (USC)

WILL YOU BE USING ANY SPECIFIC SOFTWARE? Yes. MATLAB and MAPLE.

CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

Exercises and programming assignments will contribute to 30% of the grade.

Exam which will weigh the remaining 70% of the final grade.

The teacher will personally interview students in order to evaluate the exercises and programming assignments.

CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

The same criteria as the ones used in the first assessment opportunity.

FURTHER COMMENTS:

Teachers will teach in English if need to.