Modeling and data assimilation for pre-stresses recovery in the context of structured health monitoring

Keywords: Structural health monitoring, non-destructive testing, guided-waves, data-assimilation, linear and non-linear elastodynamics, numerical analysis,

Abstract: The objective of this thesis is twofold. First, we plan to develop **accurate models and robust numerical methods** that will allow a better understanding of the physical phenomena behind the propagation of ultrasounds in stratified plates, typically used in aeronautics, that undergo large deformations. Second, to develop strategies, based on **surface or embedded measurements** that will allow the parametrization of these models and more precisely the recovery of parameters related to the integrity of the whole structures (as pre-stresses). The problems treated will concerned the topics of **numerical analysis** and **data-assimilation** as well as **mechanical and wave propagation modeling**.

Context and positioning of the thesis

The context of this thesis is the structural health monitoring (SHM) of materials. SHM is spreading across numerous industrial fields, aeronautics being one important example. This technique aims at using one or an array of actuators and sensors, embedded or onto the specimen, in order to perform *in situ* controls. Collected data are used to feed fully – or partially – automated analysis tool chains. This analysis is intended to ascertain the integrity of the structure, to evaluate its life expectancy or to adjust its maintenance cycle.

Among various possible ways to implement SHM, guided waves based SHM is receiving a significant amount of interest. The main sought advantage of this approach is to rely on the capacity of guided waves to propagate over large distances in order to broaden the size of the inspected region and to reduce the number of sensors. However, by nature, SHM is performed during exploitation, i.e. while the specimen under inspection is online. Thus, it is subject to important internal stresses, potentially inducing modifications to the way waves propagate.

The general positioning of the thesis is the modeling of wave propagation in the aforementioned context, aiming at facilitating the interpretation and analysis of the experimental data. In particular, two main issues are addressed: (1) building an efficient numerical method able to represent the complexity of guided waves propagation in pre-stressed materials; (2) proposing an inverse method based upon a data assimilation approach leading to the reconstruction of the internal stress within the specimen.

Scientific and technical description

The subject of the thesis can be separated into three *topics*.

- **Models for the propagation of waves in stratified medium undergoing large deformation**. It consists in writing a set of partial differential equations (PDE) that well describe the complex mechanical phenomena that are relevant for our study, this will define the **forward problem**. We want to show that these equations satisfy some mathematical properties, like energy balance and well-posedness, that will drive the numerical analysis. Our objective is also to characterize in details what the pre-stresses correspond to – in terms of PDE – in order to define accurately later the data-assimilation strategy.

- **Numerical analysis and scientific computing:** The mathematical models considered or derived in the previous step have to be discretized before implementation. **Space-time discretisations** – possibly with high-order finite elements – have to be developed and analyzed in terms of stability to guarantee an adequate behavior of the computations; Finally, an **implementation** of the obtained numerical scheme should be proposed, it should allows to solve efficiently the forward problem.

- **Construction and implementation of a data-assimilation strategy for the recovery of pre-stresses**. In this step our objective is to derive an **inverse problem strategy** based on data-assimilation for the reconstruction of pre-stresses. More precisely, assuming other parameters known (i.e. sources and elasticity parameters in a resting state), our objective is to define a method that will use the data available to correct the parameters corresponding to pre-stresses while they are being modified due to induced deformations.

Work-environment and required skills

The academic supervision of the thesis will be done in the team Inria-Saclay MΞDISIM jointly by P. Moireau and S. Imperiale, along with the co-supervision of A. Imperiale of CEA - LIST. The proposed thesis subject will require from the candidate to have a strong knowledge in applied mathematics and programming skills (C++ and Matlab) as well as some fundamentals in mechanics or wave propagation problems. Note that a 4 to 6 months training periods will be proposed before the beginning of the thesis. Contacts:

Philippe Moireau: `philippe.moireau@inria.fr`,

Sébastien Imperiale: `sebastien.imperiale@inria.fr`,

Alexandre Imperiale: `alexandre.imperiale@inria.fr`.